and

$$\theta = (2\lambda - 1)\frac{\pi}{2} + \frac{q}{p}(4\mu \pm 1)\frac{\pi}{2}, \qquad \left(\lambda = 1, 2, 3, \cdots, \frac{q-1}{2};\ \mu = 0, 1, 2, \cdots, p-1\right);$$

determine the same set of points on the curve

$$\rho = a \cos \frac{p}{q}\theta,$$

where p and q are two odd integers without a common factor, and a is any constant.

471. Proposed by C. N. SCHMALL, New York City.

In the ellipse $x^2/a^2 + y^2/b^2 = 1$, an *equilateral* hexagon is inscribed with two sides parallel to the major axis. In the major auxiliary circle the same thing is done. If H_1 and H_2 be the sides of the hexagons, and e the eccentricity of the ellipse, show that $H_1 : H_2 :: 4 - 2e^2 : 4 - e^2$

CALCULUS.

390. Proposed by WILSON L. MISER, University of Minnesota.

Show that the triangle whose area is a constant and whose perimeter is a minimum is equilateral.

391. Proposed by H. B. PHILLIPS, Massachusetts Institute of Technology.

If $0 < \lambda < 1$ and $0 < x < \pi$, show that the function $(\sin \lambda x)/(\sin x)$ increases as x increases.

392. Proposed by HORACE OLSON, Student at The University of Chicago.

Two right circular cylinders of radii a and b respectively, are placed so that their axes intersect at right angles. Find the volume common to them.

MECHANICS.

314. Proposed by C. N. SCHMALL, New York City.

A rectangular box of height h, and having a plane mirror for its bottom, contains a quantity of water of unknown height x. In the lid are two small apertures distant $2a$ from each other. A ray of light entering one aperture with an angle of incidence ϕ, emerges, after refraction and reflection, through the other aperture. If μ be the index of refraction of water, show that the height of the water is

$$x = \frac{(h \tan \phi - a)}{\left[\tan \phi - \dfrac{\sin \phi}{(\mu^2 - \sin^2 \phi)^{\frac{1}{2}}}\right]}.$$

SOLUTIONS OF PROBLEMS.

ALGEBRA.

427. Proposed by CLIFFORD N. MILLS, Brookings, S. Dak.

If $r \sin (\theta + \alpha) = m$, and $r \cos (\theta + \beta) = n$, show that

$$r = \frac{\sqrt{m^2 + n^2 - 2mn \sin (\alpha - \beta)}}{\cos (\alpha - \beta)}.$$

SOLUTION BY J. H. KELLOGG, Oberlin College.

From trigonometry, using only the positive square root, we know that